A Convex Relaxation Framework for Strategic Bidding in Electricity Markets

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Outline

M. Ghamkhari, A. Sadeghi-Mobarakeh, H. Mohsenian-Rad, "Strategic Bidding for Producers in Nodal Electricity Markets: A Convex Relaxation Approach," *Accepted for Publication in IEEE Transactions on Power Systems*, July 2016

Joint work with Ashkan Sadeghi-Mobarakeh and Hamed Mohsenian-Rad

History

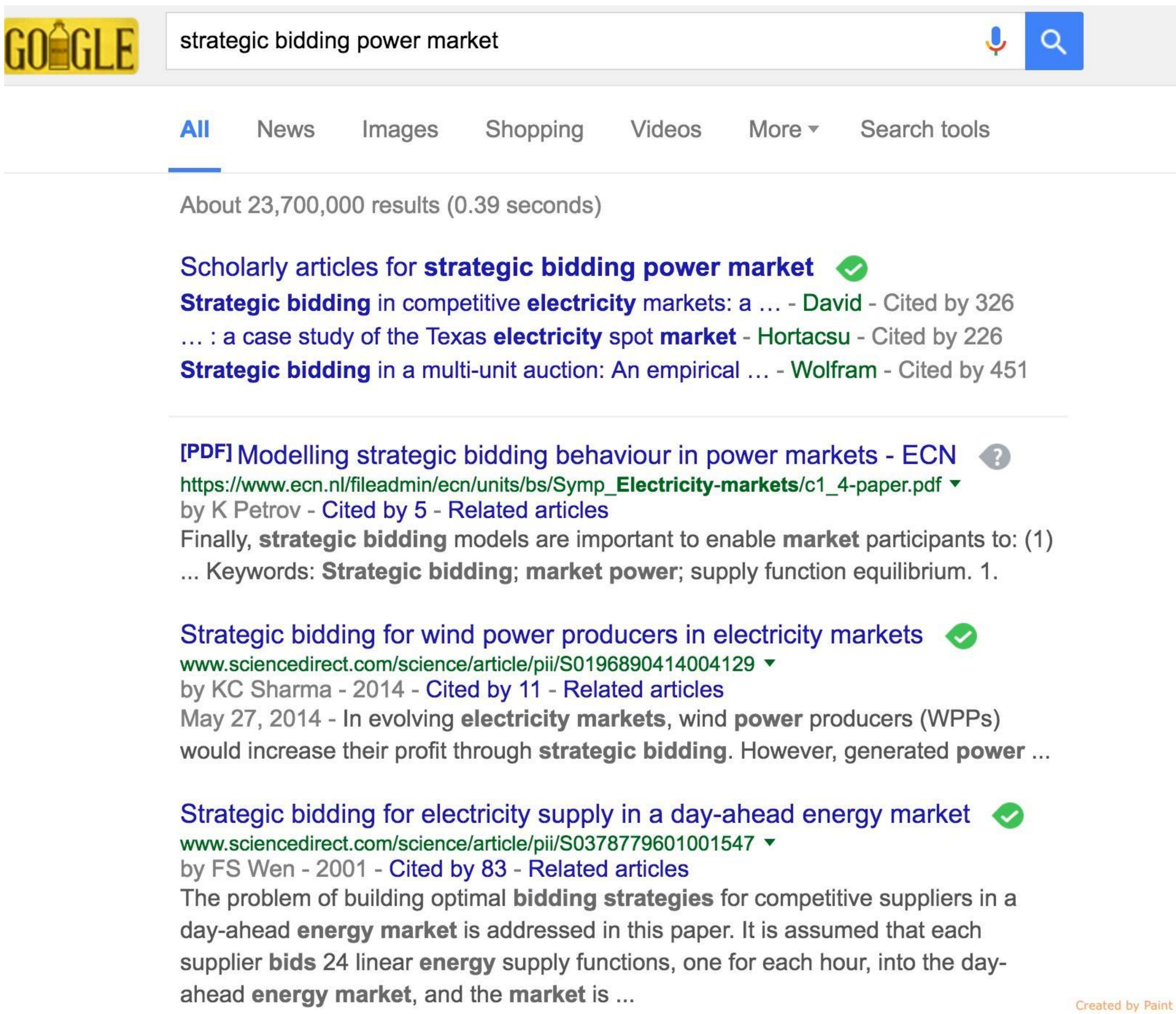
Strategic gaming analysis for electric power systems: an MPEC ...

ieeexplore.ieee.org/iel5/59/18773/00867153.pdf

by BF Hobbs - 2000 - Cited by 661 - Related articles

... **MPEC** Approach. Benjamin F. **Hobbs**, Member, IEEE, Carolyn B. Metzler, and Jong-Shi Pang .... limits the number of **bidding strategies** that can be considered.

History


GOOGLE
strategic bidding power market
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About 23,700,000 results (0.39 seconds)
Scholarly articles for strategic bidding power market
Strategic bidding in competitive electricity markets: a … - David - Cited by 326
… : a case study of the Texas electricity spot market - Hortacsu - Cited by 226
Strategic bidding in a multi-unit auction: An empirical … - Wolfram - Cited by 451
[PDF] Modelling strategic bidding behaviour in power markets - ECN
https://www.ecn.nl/fileadmin/ecn/units/bs/Symp_Electricity-markets/c1_4-paper.pdf
by K Petrov - Cited by 5 - Related articles
Finally, strategic bidding models are important to enable market participants to: (1)
... Keywords: Strategic bidding; market power; supply function equilibrium. 1.
Strategic bidding for wind power producers in electricity markets
www.sciencedirect.com/science/article/pii/S0196890414004129
by KC Sharma - 2014 - Cited by 11 - Related articles
May 27, 2014 - In evolving electricity markets, wind power producers (WPPs)
would increase their profit through strategic bidding. However, generated power ...
Strategic bidding for electricity supply in a day-ahead energy market
www.sciencedirect.com/science/article/pii/S0378779601001547
by FS Wen - 2001 - Cited by 83 - Related articles
The problem of building optimal bidding strategies for competitive suppliers in a
day-ahead energy market is addressed in this paper. It is assumed that each
supplier bids 24 linear energy supply functions, one for each hour, into the day-
ahead energy market, and the market is ...
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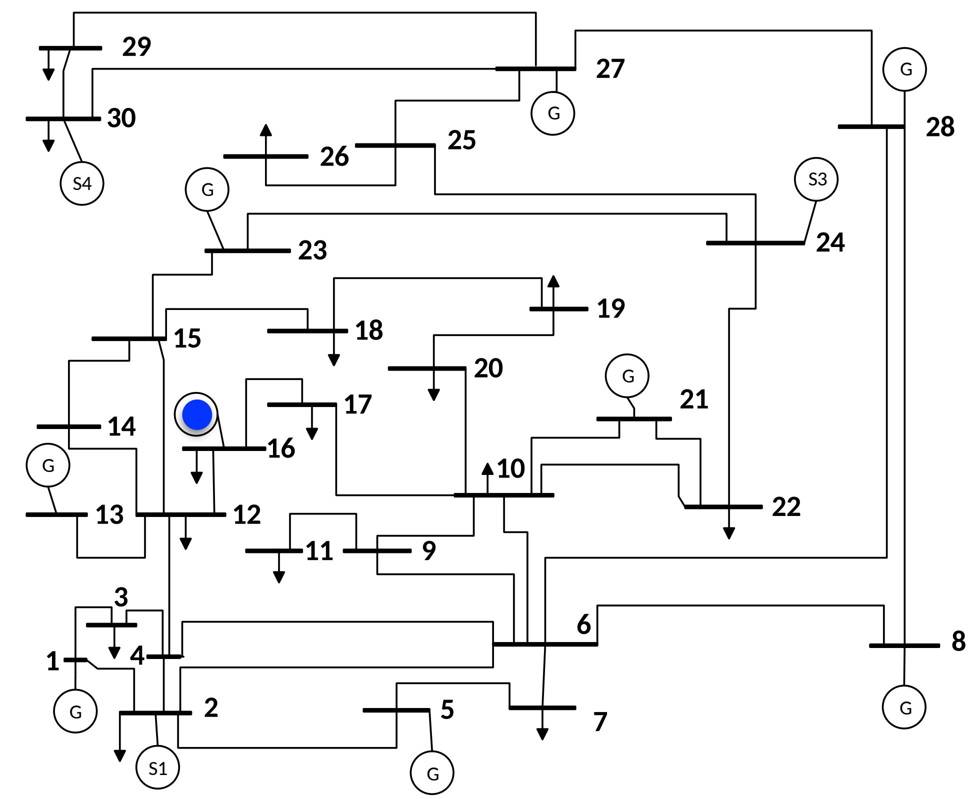
Electricity Market

Generators

Consumers

(Price, Quantity)



Electricity Network constitutes of Generators, Consumers and Transmission Lines

- Strategic Generator seeks to maximizes its profit by bidding in a strategic way

MPEC

Mathematical **P**rogram with **E**quilibrium **C**onstraints (**MPEC**)

$$Maximize\ x^T F x + 2 f^T x$$

$$p_i^T x + p_{i0} \geq 0 \qquad \forall_i$$

$$v_m^T x + v_{m0} = 0 \qquad \forall_m$$

$$x^T Q_z x + 2 q_z^T x = 0 \qquad \forall_z$$

$$Q_z = d_z q_z^T$$

Will be needed later

Inherent relation between parameters

Mixed Integer Linear Program

$$\text{Maximize } x^T F x + 2 f^T x$$

$$p_i^T x + p_{i0} \geq 0 \qquad \forall_i$$

$$v_m^T x + v_{m0} = 0 \qquad \forall_m$$

$$\Leftrightarrow \quad x^T Q_z x + 2 q_z^T x = 0 \qquad \forall_z$$

Binary Variable

$$0 \leq q_z^T x \leq Binary \times (Large\,Number)$$

$$0 \leq d_z^T x + 2 \leq 1 - Binary \times (Large\,Number)$$

Binary Variable

$$Q_z = d_z q_z^T$$

Pool Strategy of a Producer With Endogenous Formation of Locational ...
ieeexplore.ieee.org/iel5/59/5282387/05272241.pdf
by C Ruiz - 2009 - Cited by 136 - Related articles
Index Terms—Electricity **pool**, **endogenous** price **formation**,. LMP, offering **strategy** ... bought, and the hourly **locational marginal prices** (LMPs). Specifically, this ...

Solutions

- MILP: gives global solution

- MILP: Computation time increases Exponentially

Solutions

- MILP: gives global solution
  - Our Approach: gives global solution with 99% Optimality

- MILP: Computation time increases Exponentially
  - Our Approach:Computation Time increases Linearly

Our Approach

Upper Bound

$$\text{Minimize } \Lambda \;\longrightarrow\; \text{Maximize } x^T F x + 2 f^T x$$

$$p_i^T x + p_{i0} \geq 0 \qquad \forall_i$$

$$v_m^T x + v_{m0} = 0 \qquad \forall_m$$

$$x^T Q_z x + 2 q_z^T x = 0 \qquad \forall_z$$

Our Approach

Upper Bound

$$\text{Minimize } \Lambda \;\longrightarrow\; \text{Maximize } x^T F x + 2 f^T x$$

$$p_i^T x + p_{i0} \geq 0 \quad \forall_i$$

$$v_m^T x + v_{m0} = 0 \quad \forall_m$$

$$x^T Q_z x + 2 q_z^T x = 0 \quad \forall_z$$

Λ is upper bound **if and only if**

$$\Lambda - x^T F x - 2 f^T x \geq 0 \quad \textit{is positive on} \quad \left\{ \begin{array}{ll} p_i^T x + p_{i0} \geq 0 & \forall_i \\ v_m^T x + v_{m0} = 0 & \forall_m \\ x^T Q_z x + 2 q_z^T x = 0 & \forall_z \end{array} \right\}$$

Positivestellensatz

Polynomials that are positive on semi Algebraic sets

$$\Lambda - x^T F x - 2 f^T x \geq 0 \quad \textit{is positive on} \quad \left\{ \begin{array}{ll} p_i^T x + p_{i0} \geq 0 & \forall_i \\ v_m^T x + v_{m0} = 0 & \forall_m \\ x^T Q_z x + 2 q_z^T x = 0 & \forall_z \end{array} \right\}$$

Polynomial

Semi Algebraic Set

Schmudgen Positivestellensatz

$$\Lambda - x^T F x - 2 f^T x -$$

$$\sum_{i=1}^{I} \left(\begin{matrix} SOS \\ Polynomial \end{matrix} \right)(p_i^T x + p_{i0}) -$$

$$\sum_{i=1}^{I} \sum_{j=1}^{I} \left(\begin{matrix} SOS \\ Polynomial \end{matrix} \right)(p_i^T x + p_{i0})(p_j^T x + p_{j0}) -$$

$$\sum_{i=1}^{I} \sum_{j=1}^{I} \sum_{t}^{I} \left(\begin{matrix} SOS \\ Polynomial \end{matrix} \right)(p_i^T x + p_{i0})(p_i^T x + p_{i0})(p_t^T x + p_{t0}) -$$

$$\vdots$$

$$\sum_{m=1}^{M} \left(\begin{matrix} Arbitray \\ Polynomial \end{matrix} \right)(v_m^T x + v_{m0}) -$$

$$\sum_{z=1}^{Z} \left(\begin{matrix} Arbitrary \\ Polynomial \end{matrix} \right)(x^T Q_z x + 2 q_z^T x) \geq 0 \quad \forall x \in \mathbb{R}^n$$

$$\Lambda - x^T F x - 2 f^T x \geq 0 \quad \textit{is positive on} \quad \left\{ \begin{matrix} p_i^T x + p_{i0} \geq 0 & \forall_i \\ v_m^T x + v_{m0} = 0 & \forall_m \\ x^T Q_z x + 2 q_z^T x = 0 & \forall_z \end{matrix} \right\}$$

Polynomial

Semi Algebraic Set

Schmudgen Positivestellensatz

$$\Lambda - x^T F x - 2 f^T x -$$

$$\sum_{i=1}^{I} \left(\begin{matrix} SOS \\ Polynomial \end{matrix} \right) (p_i^T x + p_{i0}) -$$

$$\sum_{i=1}^{I} \sum_{j=1}^{I} \left(\begin{matrix} SOS \\ Polynomial \end{matrix} \right) (p_i^T x + p_{i0})(p_j^T x + p_{j0}) -$$

$$\sum_{i=1}^{I} \sum_{j=1}^{I} \sum_{t}^{I} \left(\begin{matrix} SOS \\ Polynomial \end{matrix} \right) (p_i^T x + p_{i0})(p_i^T x + p_{i0})(p_t^T x + p_{t0}) -$$

$$\vdots$$

$$\sum_{m=1}^{M} \left(\begin{matrix} Arbitray \\ Polynomial \end{matrix} \right) (v_m^T x + v_{m0}) -$$

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Schmudgen Positivestellensatz

$$\Lambda - x^T F x - 2 f^T x -$$

$$\sum_{i=1}^{I} \left(\begin{matrix} SOS \\ Polynomial \end{matrix} \right)(p_i^T x + p_{i0}) -$$

$$\sum_{i=1}^{I} \sum_{j=1}^{I} \left(\begin{matrix} SOS \\ Polynomial \end{matrix} \right)(p_i^T x + p_{i0})(p_j^T x + p_{j0}) -$$

$$\sum_{i=1}^{I} \sum_{j=1}^{I} \sum_{t}^{I} \left(\begin{matrix} SOS \\ Polynomial \end{matrix} \right)(p_i^T x + p_{i0})(p_i^T x + p_{i0})(p_t^T x + p_{t0}) -$$

$$\vdots$$

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Schmudgen Positivestellensatz

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$$\sum_{i=1}^{I} \left(\begin{matrix} SOS \\ Polynomial \end{matrix} \right)(p_i^T x + p_{i0}) -$$

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$$\sum_{i=1}^{I} \sum_{j=1}^{I} \sum_{t}^{I} \left(\begin{matrix} SOS \\ Polynomial \end{matrix} \right)(p_i^T x + p_{i0})(p_i^T x + p_{i0})(p_t^T x + p_{t0}) -$$

$$\vdots$$

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Schmudgen Positivestellensatz

$$\Lambda - x^T F x - 2 f^T x -$$

$$\sum_{i=1}^{I} \left(\begin{matrix}\textit{SOS} \\ \textit{Polynomial}\end{matrix}\right)(p_i^T x + p_{i0}) -$$

$$\sum_{i=1}^{I} \sum_{j=1}^{I} \left(\begin{matrix}\textit{SOS} \\ \textit{Polynomial}\end{matrix}\right)(p_i^T x + p_{i0})(p_j^T x + p_{j0}) -$$

$$\sum_{i=1}^{I} \sum_{j=1}^{I} \sum_{t}^{I} \left(\begin{matrix}\textit{SOS} \\ \textit{Polynomial}\end{matrix}\right)(p_i^T x + p_{i0})(p_i^T x + p_{i0})(p_t^T x + p_{t0}) -$$

$$\vdots$$

$$\sum_{m=1}^{M} \left(\begin{matrix}\textit{Arbitray} \\ \textit{Polynomial}\end{matrix}\right)(v_m^T x + v_{m0}) -$$

$$\sum_{z=1}^{Z} \left(\begin{matrix}\textit{Arbitrary} \\ \textit{Polynomial}\end{matrix}\right)(x^T Q_z x + 2 q_z^T x) \geq 0 \quad \forall\, x \in \mathbb{R}^n$$

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Schmudgen Positivestellensatz

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$$\sum_{i=1}^{I} \sum_{j=1}^{I} \sum_{t}^{I} \left(\begin{matrix} SOS \\ Polynomial \end{matrix} \right)(p_i^T x + p_{i0})(p_i^T x + p_{i0})(p_t^T x + p_{t0}) -$$

$$\vdots$$

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Schmudgen Positivestellensatz

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$$\sum_{i=1}^{I} \sum_{j=1}^{I} \left(\begin{matrix} SOS \\ Polynomial \end{matrix} \right)(p_i^T x + p_{i0})(p_j^T x + p_{j0}) -$$

$$\sum_{i=1}^{I} \sum_{j=1}^{I} \sum_{t}^{I} \left(\begin{matrix} SOS \\ Polynomial \end{matrix} \right)(p_i^T x + p_{i0})(p_i^T x + p_{i0})(p_t^T x + p_{t0}) -$$

$$\vdots$$

$$\sum_{m=1}^{M} \left(\begin{matrix} Arbitray \\ Polynomial \end{matrix} \right)(v_m^T x + v_{m0}) -$$

$$\sum_{z=1}^{Z} \left(\begin{matrix} Arbitrary \\ Polynomial \end{matrix} \right)(x^T Q_z x + 2 q_z^T x) \geq 0 \quad \forall\, x \in \mathbb{R}^n$$

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Schmudgen Positivestellensatz

$$\Lambda - x^T F x - 2 f^T x -$$

$$\sum_{i=1}^{I} (\text{SOS Polynomial})(p_i^T x + p_{i0}) -$$

$$\sum_{i=1}^{I} \sum_{j=1}^{I} (\text{SOS Polynomial})(p_i^T x + p_{i0})(p_j^T x + p_{j0}) -$$

$$\sum_{i=1}^{I} \sum_{j=1}^{I} \sum_{t}^{I} (\text{SOS Polynomial})(p_i^T x + p_{i0})(p_i^T x + p_{i0})(p_t^T x + p_{t0}) -$$

$$\vdots$$

$$\sum_{m=1}^{M} (\text{Arbitray Polynomial})(v_m^T x + v_{m0}) -$$

$$\sum_{z=1}^{Z} (\text{Arbitrary Polynomial})(x^T Q_z x + 2\, q_z^T x) \geq 0 \quad \forall\, x \in \mathbb{R}^n$$

Variables are polynomials

$$\Lambda - x^T F x - 2 f^T x \geq 0 \quad \textit{is positive on} \quad \begin{Bmatrix} p_i^T x + p_{i0} \geq 0 & \forall_i \\ v_m^T x + v_{m0} = 0 & \forall_m \\ x^T Q_z x + 2\, q_z^T x = 0 & \forall_z \end{Bmatrix}$$

Schmudgen Positivestellensatz

$$\Lambda - x^T F x - 2 f^T x -$$

$$\sum_{i=1}^{I} (\text{SOS Polynomial})(p_i^T x + p_{i0}) -$$

$$\sum_{i=1}^{I} \sum_{j=1}^{I} (\text{SOS Polynomial})(p_i^T x + p_{i0})(p_j^T x + p_{j0}) -$$

$$\sum_{i=1}^{I} \sum_{j=1}^{I} \sum_{t}^{I} (\text{SOS Polynomial})(p_i^T x + p_{i0})(p_i^T x + p_{i0})(p_t^T x + p_{t0}) -$$

$$(\text{SOS Polynomial}) = \sum (Polynomial)^2$$

$$\vdots$$

$$\sum_{m=1}^{M} (\text{Arbitray Polynomial})(v_m^T x + v_{m0}) -$$

$$\sum_{z=1}^{Z} (\text{Arbitrary Polynomial})(x^T Q_z x + 2 q_z^T x) \geq 0 \quad \forall x \in \mathbb{R}^n$$

Minimize Λ

$$\Lambda - x^T F x - 2 f^T x \geq 0 \quad \textit{is positive on} \quad \left\{ \begin{array}{ll} p_i^T x + p_{i0} \geq 0 & \forall_i \\ v_m^T x + v_{m0} = 0 & \forall_m \\ x^T Q_z x + 2 q_z^T x = 0 & \forall_z \end{array} \right\}$$

Schmudgen Positivestellensatz

$$\Lambda - x^T F x - 2 f^T x -$$

$$\sum_{i=1}^{I} \left(\begin{matrix} SOS \\ Polynomial \end{matrix} \right)(p_i^T x + p_{i0}) -$$

$$\sum_{i=1}^{I} \sum_{j=1}^{I} \left(\begin{matrix} SOS \\ Polynomial \end{matrix} \right)(p_i^T x + p_{i0})(p_j^T x + p_{j0}) -$$

$$\sum_{i=1}^{I} \sum_{j=1}^{I} \sum_{t}^{I} \left(\begin{matrix} SOS \\ Polynomial \end{matrix} \right)(p_i^T x + p_{i0})(p_i^T x + p_{i0})(p_t^T x + p_{t0}) -$$

$$\vdots$$

$$\sum_{m=1}^{M} \left(\begin{matrix} Arbitray \\ Polynomial \end{matrix} \right)(v_m^T x + v_{m0}) -$$

$$\sum_{z=1}^{Z} \left(\begin{matrix} Arbitrary \\ Polynomial \end{matrix} \right)(x^T Q_z x + 2 q_z^T x) \geq 0 \quad \forall x \in \mathbb{R}^n$$

Arbitrary Polynomials

$$\Lambda - x^T F x - 2 f^T x \geq 0 \quad \textit{is positive on} \quad \left\{ \begin{matrix} p_i^T x + p_{i0} \geq 0 & \forall_i \\ v_m^T x + v_{m0} = 0 & \forall_m \\ x^T Q_z x + 2 q_z^T x = 0 & \forall_z \end{matrix} \right\}$$

Schmudgen Positivestellensatz

$$\Lambda - x^T F x - 2 f^T x -$$

$$\sum_{i=1}^{I} \left(\text{SOS Polynomial}\right)(p_i^T x + p_{i0}) -$$

$$\sum_{i=1}^{I} \sum_{j=1}^{I} \left(\text{SOS Polynomial}\right)(p_i^T x + p_{i0})(p_j^T x + p_{j0}) -$$

$$\sum_{i=1}^{I} \sum_{j=1}^{I} \sum_{t}^{I} \left(\text{SOS Polynomial}\right)(p_i^T x + p_{i0})(p_i^T x + p_{i0})(p_t^T x + p_{t0}) -$$

$$\vdots$$

$$\sum_{m=1}^{M} \left(\text{Arbitray Polynomial}\right)(v_m^T x + v_{m0}) -$$

$$\sum_{z=1}^{Z} \left(\text{Arbitrary Polynomial}\right)(x^T Q_z x + 2 q_z^T x) \geq 0 \quad \forall\, x \in \mathbb{R}^n$$

$$\left(\text{SOS Polynomial}\right) = \sum (Polynomial)^2$$

x is not variables
x is an index for infinite constraints

$$\Lambda - x^T F x - 2 f^T x \geq 0 \quad \textit{is positive on} \quad \left\{ \begin{array}{ll} p_i^T x + p_{i0} \geq 0 & \forall_i \\ v_m^T x + v_{m0} = 0 & \forall_m \\ x^T Q_z x + 2 q_z^T x = 0 & \forall_z \end{array} \right\}$$

A Convex Optimization Problem

$Minimize \ \ \Lambda$

$$\Lambda - x^T F x - 2 f^T x -$$

$$\sum_{i=1}^{I} \left(\begin{matrix} SOS \\ Polynomial \end{matrix} \right)(p_i^T x + p_{i0}) -$$

$$\sum_{i=1}^{I} \sum_{j=1}^{I} \left(\begin{matrix} SOS \\ Polynomial \end{matrix} \right)(p_i^T x + p_{i0})(p_j^T x + p_{j0}) -$$

$$\sum_{i=1}^{I} \sum_{j=1}^{I} \sum_{t}^{I} \left(\begin{matrix} SOS \\ Polynomial \end{matrix} \right)(p_i^T x + p_{i0})(p_i^T x + p_{i0})(p_t^T x + p_{t0}) -$$

$$\vdots$$

$$\sum_{m=1}^{M} \left(\begin{matrix} Arbitray \\ Polynomial \end{matrix} \right)(v_m^T x + v_{m0}) -$$

$$\sum_{z=1}^{Z} \left(\begin{matrix} Arbitrary \\ Polynomial \end{matrix} \right)(x^T Q_z x + 2 q_z^T x) \geq 0 \quad \forall x \in \mathbb{R}^n$$

Relaxation of Polynomials in Psatz

$$\Lambda - x^T F x - 2 f^T x -$$

$$\sum_{i=1}^{I} \left(\begin{matrix} Positive \\ Scalar \end{matrix} \right) (p_i^T x + p_{i0}) -$$

$$\sum_{i=1}^{I} \sum_{j=1}^{I} \left(\begin{matrix} Positive \\ Scaler \end{matrix} \right) (p_i^T x + p_{i0})(p_j^T x + p_{j0}) -$$

$$\sum_{m=1}^{M} \left(\begin{matrix} Linear \\ Polynomial \end{matrix} \right) (v_m^T x + v_{m0}) -$$

$$\underbrace{\sum_{z=1}^{Z} \left(\begin{matrix} Arbitrary \\ Scalar \end{matrix} \right) (x^T Q_z x + 2 q_z^T x)}_{Quadratic\ Expression} \geq 0 \quad \forall x \in \mathbb{R}^n$$

Computationally Tractable Reformulation

$$\Lambda - x^T F x - 2 f^T x -$$
$$\sum_{i=1}^{I} \left(\begin{matrix} Positive \\ Scalar \end{matrix} \right) (p_i^T x + p_{i0}) -$$
$$\sum_{i=1}^{I} \sum_{j=1}^{I} \left(\begin{matrix} Positive \\ Scaler \end{matrix} \right) (p_i^T x + p_{i0})(p_j^T x + p_{j0}) -$$
$$\sum_{m=1}^{M} \left(\begin{matrix} Linear \\ Polynomial \end{matrix} \right) (v_m^T x + v_{m0}) -$$
$$\sum_{z=1}^{Z} \left(\begin{matrix} Arbitrary \\ Scalar \end{matrix} \right) (x^T Q_z x + 2 q_z^T x) \geq 0 \quad \forall x \in \mathbb{R}^n$$

Quadratic Expression

$$\begin{bmatrix} 1 \\ x \end{bmatrix}^T \gamma \begin{bmatrix} 1 \\ x \end{bmatrix} \geq 0$$

$$\gamma = \begin{pmatrix} \Lambda & f^T \\ -f & F \end{pmatrix} -$$
$$\sum_{i=1}^{I} \left(\begin{matrix} Positive \\ Scalar \end{matrix} \right) \begin{pmatrix} \underline{p_{i0}} & \underline{\frac{p_i^T}{2}} \\ \underline{\frac{p_i}{2}} & \underline{0} \end{pmatrix} -$$
$$\sum_{i=1}^{I} \sum_{j=1}^{I} \left(\begin{matrix} Positive \\ Scaler \end{matrix} \right) \begin{bmatrix} \underline{p_{j0}} \\ \underline{\frac{p_j}{2}} \end{bmatrix} \begin{bmatrix} \underline{p_{j0}} \\ \underline{\frac{p_j}{2}} \end{bmatrix}^T -$$
$$\sum_{m=1}^{M} Scalar \begin{bmatrix} \underline{1} \\ \underline{0} \end{bmatrix} \begin{bmatrix} \underline{v_{m0}} \\ \underline{\frac{v_m}{2}} \end{bmatrix}^T - \sum_{m=1}^{M} \sum_{l=1}^{n} Scalar \begin{bmatrix} \underline{0} \\ \underline{e_l} \end{bmatrix} \begin{bmatrix} \underline{v_{m0}} \\ \underline{\frac{v_m}{2}} \end{bmatrix}^T$$
$$\sum_{z=1}^{Z} \left(\begin{matrix} Positive \\ Scalar \end{matrix} \right) \begin{pmatrix} \underline{0} & \underline{q_i^T} \\ \underline{q_i} & \underline{Q_z} \end{pmatrix}$$

Computationally Tractable

$$\begin{bmatrix} 1 \\ x \end{bmatrix}^T \gamma \begin{bmatrix} 1 \\ x \end{bmatrix} \geq 0 \quad \rightleftarrows \quad \gamma \succeq 0$$

Semi Definite

Computationally Tractable

Λ is upper bound if $\gamma \succeq 0$

$$\begin{bmatrix} 1 \\ x \end{bmatrix}^T \gamma \begin{bmatrix} 1 \\ x \end{bmatrix} \geq 0 \quad \rightleftarrows \quad \gamma \succeq 0$$

Semi Definite

Computationally Tractable

$$Minimize\ \Lambda$$
$$\gamma \succeq 0$$

Best customized upper bound

$$Maximize\ x^T F x + 2 f^T x$$
$$p_i^T x + p_{i0} \geq 0 \quad \forall_i$$
$$v_m^T x + v_{m0} = 0 \quad \forall_m$$
$$x^T Q_z x + 2 q_z^T x = 0 \quad \forall_z$$

Λ is upper bound if $\gamma \succeq 0$

$$\begin{bmatrix} 1 \\ x \end{bmatrix}^T \gamma \begin{bmatrix} 1 \\ x \end{bmatrix} \geq 0 \quad \rightleftarrows \quad \gamma \succeq 0$$

Semi Definite

Computationally Tractable

$$Minimize\ \Lambda$$
$$\gamma \succeq 0$$

Λ is 97% optimal

Computationally Tractable

$$\mathit{Minimize}\ \Lambda$$

$$\gamma \succeq 0$$

Λ is 97% optimal

$$\begin{bmatrix} 1 \\ x \end{bmatrix}^T \gamma \begin{bmatrix} 1 \\ x \end{bmatrix} \geq 0 \rightleftarrows \gamma \succeq 0$$

What happened to x?

x: Optimal optimization variables in MPEC ?

Recovery

$$Minimize\ \Lambda$$
$$\gamma \succeq 0$$

Duality

Dual Form PrimalForm

$$\text{maximize } trace\left(\begin{pmatrix} 0 & f^T \\ f & F \end{pmatrix} X\right)$$

$$trace\left(\begin{pmatrix} \underline{p_{i0}} & \frac{p_i^T}{2} \\ \frac{p_i}{2} & \underline{0} \end{pmatrix} X\right) \geq 0 \quad \forall\ i$$

$$trace\left(e_l \begin{bmatrix} v_{m0} \\ v_m \end{bmatrix}^T X\right) = 0 \quad \forall\ m, l$$

$$trace\left(\begin{bmatrix} p_{i0} \\ p_i \end{bmatrix}\begin{bmatrix} p_{j0} \\ p_j \end{bmatrix}^T X\right) \geq 0 \quad \forall\ i\ j$$

$$trace\left(\begin{pmatrix} \underline{0} & \underline{q_z^T} \\ \underline{q_z} & \underline{Q_z} \end{pmatrix} X\right) \geq 0 \quad \forall\ z$$

$$X_{11} = 1$$
$$X \succeq 0$$

Recovery

Approximated solution for variables in MPEC

$$\begin{bmatrix} 1 \\ x^* \end{bmatrix} = X^* \begin{bmatrix} 1 \\ 0 \end{bmatrix}^T$$

First column of X

$$\text{maximize } trace\left(\begin{pmatrix} 0 & f^T \\ f & F \end{pmatrix} X\right)$$

$$trace\left(\begin{pmatrix} p_{i0} & \frac{p_i^T}{2} \\ \frac{p_i}{2} & 0 \end{pmatrix} X\right) \geq 0 \quad \forall\, i$$

$$trace\left(e_l \begin{bmatrix} v_{m0} \\ v_m \end{bmatrix}^T X\right) = 0 \quad \forall\, m, l$$

$$trace\left(\begin{bmatrix} p_{i0} \\ p_i \end{bmatrix} \begin{bmatrix} p_{j0} \\ p_j \end{bmatrix}^T X\right) \geq 0 \quad \forall\, i\ j$$

$$trace\left(\begin{pmatrix} 0 & q_z^T \\ q_z & Q_z \end{pmatrix} X\right) \geq 0 \quad \forall\, z$$

$$X_{11} = 1$$

$$X \succeq 0$$

Recovery

Approximated solution
for variables in MPEC

$$\begin{bmatrix} 1 \\ x^* \end{bmatrix} = X^* \begin{bmatrix} 1 \\ 0 \end{bmatrix}^T$$

First column of X

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$$trace\left(\begin{pmatrix} p_{i0} & \frac{p_i^T}{2} \\ \frac{p_i}{2} & 0 \end{pmatrix} X\right) \geq 0 \quad \forall\, i$$

$$trace\left(e_l \begin{bmatrix} v_{m0} \\ v_m \end{bmatrix}^T X\right) = 0 \quad \forall\, m, l$$

$$trace\left(\begin{bmatrix} p_{i0} \\ p_i \end{bmatrix} \begin{bmatrix} p_{j0} \\ p_j \end{bmatrix}^T X\right) \geq 0 \quad \forall\, i, j$$

$$trace\left(\begin{pmatrix} 0 & q_z^T \\ q_z & Q_z \end{pmatrix} X\right) \geq 0 \quad \forall\, z$$

$$X_{11} = 1$$

$$X \succeq 0$$

X^* is not feasible in MPEC

Recovery

Approximated solution for variables in MPEC

$$\begin{bmatrix} 1 \\ x^* \end{bmatrix} = X^* \begin{bmatrix} 1 \\ 0 \end{bmatrix}^T$$

we use x^* to produce a feasible solution to MPEC

$$\text{Maximize } x^T F x + 2 f^T x$$

$$p_i^T x + p_{i0} \geq 0 \quad \forall_i$$

$$v_m^T x + v_{m0} = 0 \quad \forall_m$$

$$x^T Q_z x + 2 q_z^T x = 0 \quad \forall_z$$

$$Q_z = d_z q_z^T$$

Recovery

Approximated solution for variables in MPEC

$$\begin{bmatrix} 1 \\ x^* \end{bmatrix} = X^* \begin{bmatrix} 1 \\ 0 \end{bmatrix}^T$$

we use X^* to produce a feasible solution to MPEC

$$Maximize \; x^T F x + 2 f^T x$$

$$p_i^T x + p_{i0} \geq 0 \qquad \forall_i$$

$$v_m^T x + v_{m0} = 0 \qquad \forall_m$$

$$x^T Q_z x + 2 q_z^T x = 0 \qquad \forall_z$$

$$Q_z = d_z q_z^T$$

$$\left(d_z^T x + 2\right)\left(q_z^T x\right) = 0$$

Recovery

Approximated solution for variables in MPEC

$$\begin{bmatrix} 1 \\ x^* \end{bmatrix} = X^* \begin{bmatrix} 1 \\ 0 \end{bmatrix}^T$$

we use x^* to produce a feasible solution to MPEC

$$\left| d_z^T x^* + 2 \right| \leq \varepsilon \quad \textbf{and} \quad \left| q_z^T x^* \right| \geq \Delta \quad \Rightarrow \quad \left(d_z^T x + 2 \right) = 0$$

$$\left(d_z^T x + 2 \right)\left(q_z^T x \right) = 0$$

$$\left(d_z^T x + 2 \right) = 0$$

$$\left(q_z^T x \right) = 0$$

Recovery

Approximated solution for variables in MPEC

$$\begin{bmatrix} 1 \\ x^* \end{bmatrix} = X^* \begin{bmatrix} 1 \\ 0 \end{bmatrix}^T$$

we use x^* to produce a feasible solution to MPEC

$$\left(d_z^T x + 2\right)\left(q_z^T x\right) = 0 \quad \Rightarrow \quad \left(d_z^T x + 2\right) = 0 \quad \text{or} \quad \left(q_z^T x\right) = 0$$

$\left|d_z^T x^* + 2\right| \geq \Delta$ **and** $\left|q_z^T x^*\right| \leq \varepsilon$ $\Rightarrow$ $\left(q_z^T x\right) = 0$

Recovery

Approximated solution for variables in MPEC

$$\begin{bmatrix} 1 \\ x^* \end{bmatrix} = X^* \begin{bmatrix} 1 \\ 0 \end{bmatrix}^T$$

we use X^* to produce a feasible solution to MPEC

$$Maximize\ x^T F x + 2 f^T x$$

$$p_i^T x + p_{i0} \geq 0 \qquad \forall_i$$

$$v_m^T x + v_{m0} = 0 \qquad \forall_m$$

$$x^T Q_z x + 2 q_z^T x = 0 \qquad \forall_z$$

Algorithm

$$\text{maximize } trace\left(\begin{pmatrix} 0 & f^T \\ f & F \end{pmatrix} X\right)$$

$$trace\left(\begin{pmatrix} \underline{p_{i0}} & \frac{p_i^T}{2} \\ \frac{p_i}{2} & \underline{0} \end{pmatrix} X\right) \geq 0 \quad \forall i$$

$$trace\left(e_l \begin{bmatrix} v_{m0} \\ v_m \end{bmatrix}^T X\right) = 0 \quad \forall m \cdot l$$

$$trace\left(\begin{bmatrix} p_{i0} \\ p_i \end{bmatrix}\begin{bmatrix} p_{j0} \\ p_j \end{bmatrix}^T X\right) \geq 0 \quad \forall i \ j$$

$$trace\left(\begin{pmatrix} \underline{0} & q_z^T \\ q_z & Q_z \end{pmatrix} X\right) \geq 0 \quad \forall z$$

$$X_{11} = 1$$

$$X \succeq 0$$

1. Solve the dual of customized Psatz Relaxation

2. Obtain an approximated solution for MPEC

$$\begin{bmatrix} 1 \\ x^* \end{bmatrix} = X^* \begin{bmatrix} 1 \\ 0 \end{bmatrix}^T$$

3. Eliminate some non-convex constraints

$$\left(d_z^T x + 2\right)\left(q_z^T x\right) = 0 \;\rightarrow\; \left(d_z^T x + 2\right) = 0 \quad \text{or} \quad \left(q_z^T x\right) = 0$$

4. Solve a reduced complexity MILP

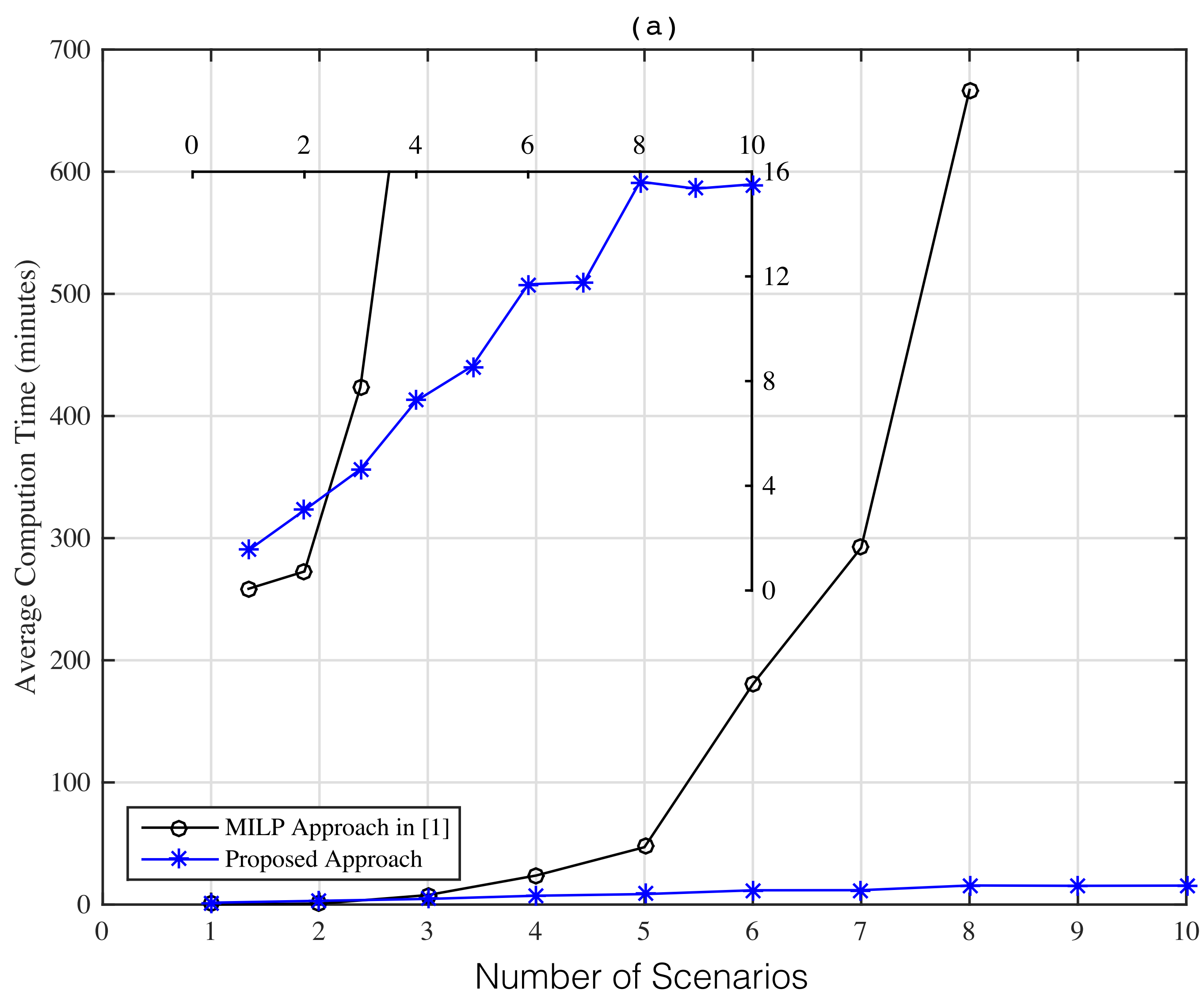
$$Maximize \ x^T F x + 2 f^T x$$

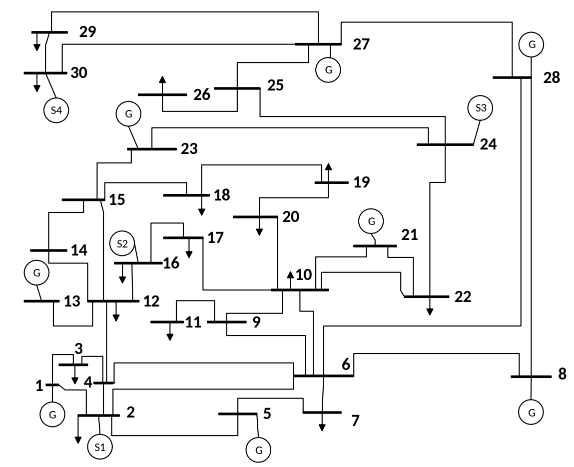
$$p_i^T x + p_{i0} \geq 0 \qquad \forall_i$$

$$v_m^T x + v_{m0} = 0 \qquad \forall_m$$

$$x^T Q_z x + 2 q_z^T x = 0 \qquad \forall_z$$

Simulation Results

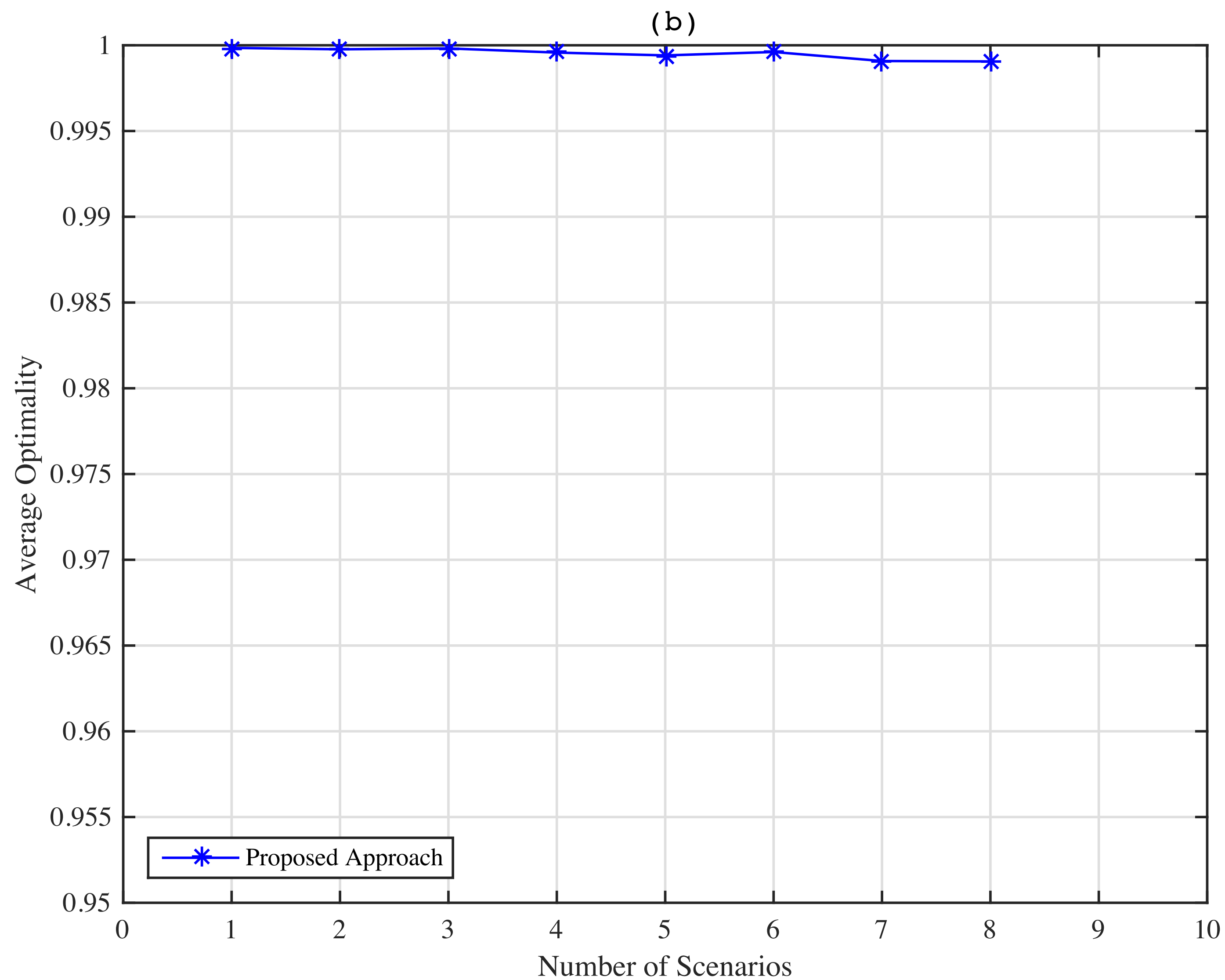


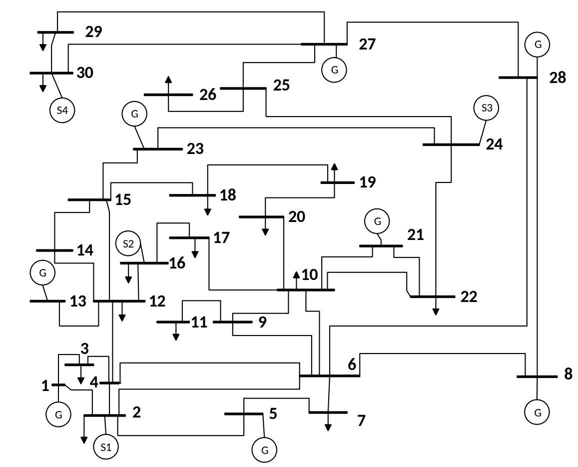


IEEE 30-Bus Network

The impact of increasing the number of random scenarios on the computation time of the proposed approach and the MILP approach

Simulation Results





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The impact of increasing the number of random scenarios on the optimality of the proposed approach.

Thank You